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Erratum: Quantum propagation of neutral atoms in a magnetic quadrupole guide [Phys. Rev. A 61, 033614 (2000)]

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The quantum number ℓ in the wave function $\psi(\rho,\phi)$ is in fact not an eigenvalue of the orbital angular momentum L_z of the motion of the atom around the center of the guide, as we had mistakenly assumed without checking, but an eigenvalue of the operator $L_z - s_z$, where s_z is the z component of the total internal angular momentum of the atom that couples to the magnetic field. For a magnetic quadrupole field of the form of Eq. (1.1), $\mathbf{B}=(4B_0x/R,-4B_0y/R,0)$ the operators L_z and s_z do not individually commute with the Hamiltonian $H=\mathbf{p}^2/(2m)+g\mu_B\mathbf{s}\cdot\mathbf{B}$,

$$[L_z,H]=ig\mu_B\hbar\frac{4B_0}{R}(s_yx+s_xy),$$

$$[s_z,H]=ig\mu_B\hbar\frac{4B_0}{R}(s_yx+s_xy),$$

but the difference L_z-s_z commutes with H and is thus a conserved quantity, $[L_z-s_z,H]=0$. Calculation shows that the quantum number ℓ in the wave function $\psi(\rho,\phi)$ is the eigenvalue of L_z-s_z, e.g., for spin 1/2

$$(L_z-s_z)\frac{1}{\sqrt{2}}\begin{pmatrix}F_+(\rho)e^{i(\ell+1/2)\phi}\\F_-(\rho)e^{i(\ell-1/2)\phi}\end{pmatrix}=\frac{\hbar}{\sqrt{2}}\begin{pmatrix}(\ell+1/2)F_+(\rho)e^{i(\ell+1/2)\phi}\\(\ell-1/2)F_-(\rho)e^{i(\ell-1/2)\phi}\end{pmatrix}-\frac{\hbar}{2\sqrt{2}}\begin{pmatrix}F_+(\rho)e^{i(\ell+1/2)\phi}\\-F_-(\rho)e^{i(\ell-1/2)\phi}\end{pmatrix}$$

$$=\ell\frac{\hbar}{\sqrt{2}}\begin{pmatrix}F_+(\rho)e^{i(\ell+1/2)\phi}\\F_-(\rho)e^{i(\ell-1/2)\phi}\end{pmatrix},$$

and similarly for the spin 1 wave function in Eq. (4.5). Although L_z and s_z do not commute with H, they commute with L_z-s_z and with each other, which means that any eigenstate of L_z-s_z must be a linear combination of simultaneous eigenstates of L_z and s_z. Since the eigenvalues of L_z are always integer, it follows that the eigenvalue ℓ of L_z-s_z must be half integer for half integer spin **s** and integer for integer spin **s**. Only then does the wave function have the correct transformation properties under rotations. For spin 1/2 a rotation by 2π around the z axis, for example, transforms the wave function of the above eigenstate into

$$\frac{1}{\sqrt{2}}F_+(\rho)e^{i(\ell+1/2)(\phi+2\pi)}\mathcal{D}^{1/2}_{mm'}(0,0,2\pi)\begin{pmatrix}1\\0\end{pmatrix}+\frac{1}{\sqrt{2}}F_-(\rho)e^{i(\ell-1/2)(\phi+2\pi)}\mathcal{D}^{1/2}_{mm'}(0,0,2\pi)\begin{pmatrix}0\\1\end{pmatrix}.$$

Since the application of the spin rotation matrix $\mathcal{D}^{1/2}_{mm'}(0,0,2\pi)$ causes the spinors to change sign, the ϕ dependent prefactors must not change sign under $\phi\rightarrow\phi+2\pi$. Thus ℓ has to be half integer for spin 1/2, and not integer as was erroneously stated in Sec. III. It follows that ℓ cannot be zero, which, as explained by Eq. (5.3), has the consequence that there are no exact bound states in the case of spin 1/2 and makes Sec. III A redundant. Solving Eqs. (3.11) for $\ell=1/2,3/2,\ldots$ gives results that are qualitatively the same as those of Sec. III B. For $\ell=1/2$ the energies and widths (ϵ_i,Γ_i) of the first three resonances are (2.64, 0.34), (4.25, 0.34), and (5.62, 0.34), and for $\ell=3/2$ they are (3.53, 0.11), (5.04, 0.15), and (6.34, 0.18), in units of $(\hbar^2G^2/2m)^{1/3}$. The similarity of the motion of spin 1/2 and of spin 1 atoms in the guide is underlined by the fact that Eqs. (3.7) for spin 1/2 and $\ell=1/2$ are structurally the same as Eqs. (4.7a) and (4.7b) for spin 1 and $\ell=0$ except for a replacement of G by $2G$.

The energies and widths for $s=1/2$ and the unphysical case of integer ℓ have been reproduced by a complex scaling calculation by Potvliege and Zehnlé [1]. These authors agree that ℓ should in fact be taken half integer [2]. The applicability of their method is independent of whether integer or half integer values of ℓ are being considered.

Nothing changes for spin 1 in Sec. IV.

There are also two misprints. Contrary to what is stated in the text of Sec. II, we have used $G=2g\mu_BB_0/R$ throughout the paper. The last paragraph of Sec. VII discusses the first excitation energy for an $\ell=0$ atom of ^{87}Rb, and not $\ell=1$ as originally stated.

We are indebted to Dr. John Stockton, Dr. Clifford Hicks, and Professor Hideo Mabuchi for pointing out to us that ℓ is an eigenvalue of L_z-s_z and must be half integer for spin 1/2.

[1] R.M. Potvliege and V. Zehnlé, Phys. Rev. A **63**, 025601 (2001).

[2] R. M. Potvliege, private communication.

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